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Investigating the Relationship between Slopes of Roofs and Maximum Live Load

David J. Kim

Gyeonggi Suwon International School, Suwon, Republic of Korea

Abstract: **A roof is structured to support its own weight —the dead load— and the weight of temporary objects resting on it —the live load. The purpose of this investigation was to determine the optimal slope of roof for it to support the most live load. With the use of roof structure models and weights, the ability of the roof to withstand mass was investigated.**

Roof structure models of particular slopes —0°, 15°, 30°, 45°, and 60°— were created using thin strips of pinewood and superglue. Sand was added to the plastic bucket that was tied to a nylon string hung over the model until the model broke. The mass of the plastic bucket filled with sand was then measured and recorded as the break mass. The break masses were averaged and converted to *psf (pounds per square feet)*, a widely used unit for live load of real roofs.

According to several articles and academic journals, the maximum live load decreases as the slope of roof decreases, indicating that a flat roof, a roof of slope 0°, is the optimal slope. The hypothesis was partly justified as the experiment failed to show that the flat roof supported the most live load while still managing to justify that maximum live load decreases as slope of the roof increases as the average maximum live load of 15°, 30°, 45°, and 60° decreased respectively. Hence, coming to a conclusion that maximum live load decreases as slope of roof increases after 0°.

Keywords: **Break Mass, Dead Load, Slope of Roof, Maximum Live Load, Optimal Slope, and Roof Structure.**

I. INTRODUCTION

Due to technological advancements, the construction time of buildings has significantly reduced. Nowadays, a 30-story building can be built in 15 days [4]. With continuous development of technology, the demand for even faster constructions has increased notably. Consequently, the construction industries all compete for speed. A fast construction is beneficial in that it allows huge, complex buildings, towers, skyscrapers, and infrastructures to be built quickly. However, often times the extremely fast-paced construction leads to flaws and errors in buildings [10].

Such is the case of the collapsing roof of a resort in Korea that had been accommodating hundreds of college students. The roof of the auditorium building at the resort collapsed when it failed to hold up the weight of the snow, trapping hundreds of students under the snow and metal roof supports [6].

The roof is supposed to be constructed in a way so that it can support the weight of itself as well as the weight that rests on top of it. To put it in a more architectural terminology, the roof has to support both the dead load —it's own weight— as well as the live load —weight of temporary objects resting on it. The amount of dead load and live load a roof can handle depends on how well the roof is able to distribute the force exerted on a particular area through the supporting walls and ultimately to the ground [3]. The roof of the auditorium at the resort was capable of supporting the dead load; but when the live load exceeded a certain point, it collapsed. Roofs come in various shapes and inclines, and the particular roof that collapsed in Korea was a flat roof [5]. If the roof had been an inclined roof, would it not have collapsed? How significant is the slope of the roof in its ability to withstand mass?

Thus, it seemed reasonable that there is an optimal slope for roofs that can withstand the most loads. Therefore, in order to determine the slope of the roof that can withstand the most load, the optimal slope of roof, I came up with the following research question: How does the slope of roof affect its maximum live load?

The amount of load supported by a roof is calculated through adding both of its dead load and live load. Both of these numbers are often represented as pounds per square foot, psf. It expresses that a square foot of area can withstand up to certain amount of load in pounds [7]. For instance, a roof with a 15 psf dead load is capable of supporting 15 pounds of itself per square foot. If the same roof has a live load of 20 psf means that it is capable of supporting 20 pounds of any object resting on top per square foot. Thus, the total load of this particular roof will be 15 psf + 20 psf = 35 psf [9].

The live load of a roof depends significantly on the slope of the roof. A flat roof distributes the force from the load horizontally while an inclined roof distributes the force from the load at an angle. According to several articles and roof live load calculations, as the slope of the roof increases, the amount of load it can withstand decreases, concluding that a flat roof will support the most live load [2]. This investigation will find out whether this hypothesis is true through experimentation and justify the results through physics principles.

II. EXPERIMENTAL DESIGN

A. Initial Experimental Design:

Prior to coming up with the final design of the experiment, the entire roof frame model including rafters, studs, ridge board, wall plate, and ceiling joist (see Appendix) was intended to be created instead of using just one section of the roof frame. The roof frame model with various slopes was to rest on top of four 15 cm high posts placed at each corners of a 15 cm by 15 cm square and a nylon string with lab weights tied to its ends was to be hung over the center of the ridge board. The weights were then to be added to the ends of the nylon string until the roof frame model broke. The break mass of each roof frame models were to be recorded, and by observing the change in break mass of roof frame models with various slopes, the experiment hoped to determine a relationship between the slope of the roof and its maximum live load.

However, major flaws were evident in this design. First, the force acting on the ridge board was mostly distributed through the studs of the roof and not through the sloped rafters. Such distribution of force through the stud puts most of the stress on the ridge board. A test trial was done and only the ridge board was bending. The ridge board eventually broke due to too much weight, surpassing its maximum live load. The following design then would find the relationship between the ridge board and the ridge board's maximum live load instead of the relationship between the slope of the roof and the sloped roof's maximum live load.

Second, because the weights had a set mass, the increasing interval of weights was too big. For instance, if the roof structure held 1 kg, the next trial would then have to be 1.5 kg due to the weights available. This poses a problem to the measurement. If the structure holds 1 kg but breaks at 1.5 kg, then the measurement of the break mass becomes inaccurate. The structure could have held 1.1 kg or even 1.27 kg but because the interval was in 0.5 kg, the break mass becomes significantly inaccurate with 1.5 kg.

Lastly, the time required to create each roof structure models was very time consuming. Creating just one roof structure model for the test run required a lot of material and drying time. Doing 10 trials for 5 different sloped roofs meant that a total of at least 50 roof structure models were needed. With the amount of material and time required to create just one roof structure, creating 50 was challenging.

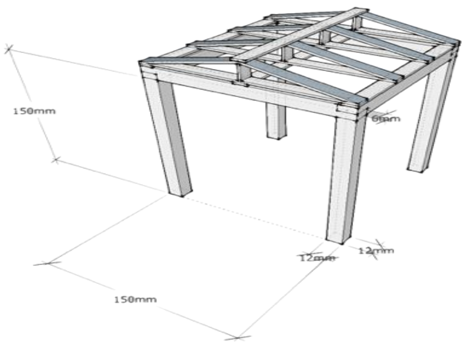

Fig. 1: Initial experiment design on top of four 15 cm high posts
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Fig. 2: Initial roof frame model with rafters, studs, ridge board, and ceiling joists

B. Final Experimental Design:

To compensate for the flaws above, the design was modified. Instead of using an entire roof frame model, using only one section of the roof frame was more effective and appropriate to investigate the relationship between slope of the roof and its maximum live load.

Using a section of the roof frame was possible because in real roof frames, the force applied to the roof is eventually distributed through each frame. Hence, experimenting with just a section of the roof frame did not change any conditions except reducing the number of roof frames. By doing so, it solved all the major flaws. Using a section of the roof frame enabled the weight to be stressed on the sloped rafters rather than the ridge board and the studs. The nylon string no longer hangs over the ridge board but the rafter. This enables the force to be distributed through the rafters. A test trial also showed that now the rafter broke instead of the ridge board. Thus, the force now properly acts on the rafters to show the relationship between slope of the roof and its maximum live load.

Buckets and sand also accompanied weights. Instead of tying the weights directly to the nylon string, a bucket was tied to the ends. Weights were placed in the bucket. The roof structure model easily held 2 kg, so two 1 kg weights were placed at the ends of the bucket. Next, a smaller bucket was placed on the existing bucket. Sand was poured until the structure broke. The mass of the entire big bucket with weights, small bucket and sand were measured to determine the break mass of the structures. Sand was used because it weighs significantly little. The increasing interval of the load is then much smaller, allowing a much more accurate measurement.

Lastly, because only a section of the roof frame is needed instead of the entire roof frame, the time needed to create the structures significantly reduced. Now, four roof frame sections can be created with the material and time it takes to make one whole roof frame. As a result, the final experimental design bettered the experiment in all aspects.

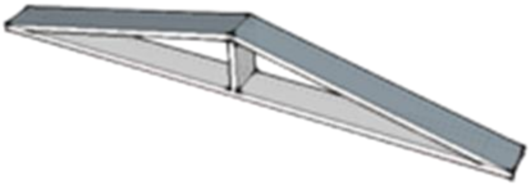


Fig. 3: Final roof structure model, which is only a section of the initial roof frame

III. METHOD

A. Method of Creating Roof Structure Model:

i. Materials Needed: Listed are materials needed to create the roof structure models

- 3 Pine Wood Strips: 90 cm long, 1 cm wide, and 0.1 cm thick
- Superglue
- Ruler
- Knife
- Coping Saw
- Sandpaper

ii. Method:

a. 0°

1. Gather all materials
2. Measure 15 cm of pinewood strip
3. Cut pinewood strips into 15 cm piece with the coping saw
4. Sandpaper all the edges of the wood pieces
5. Mark the midpoint of the 15 cm piece with a knife, creating a very thin groove
6. Repeat steps 2 to 5 until 10 pieces are prepared

b. 15°, 30°, 45°, 60°

1. Gather all materials
2. Measure 15 cm of pinewood strip
3. Cut pinewood strips into 15 cm piece with the coping saw
4. Measure 2 cm of pinewood strip
5. Cut pinewood strips into 2 cm piece with the coping saw
6. Measure two 8 cm of pinewood strips
7. Cut pinewood strips into two 8 cm pieces with the coping saw
8. Mark the midpoint of one 8 cm piece with a knife, creating a very thin groove
9. Sandpaper all the edges of the wood pieces
10. Use superglue to glue the 2 cm piece vertically to the midpoint of the 15 cm piece
11. Let it dry until the two are firmly glued together
12. Use superglue to glue the two 8 cm pieces slanted onto the endpoints of the 2 cm and 15 cm pieces, making two right triangles
13. Repeat steps 2 to 12 until 10 pieces are prepared
14. Repeat steps 2 to 13 using 4 cm and 9 cm in place of 2 cm and 8 cm respectively for 30°
15. Repeat steps 2 to 13 using 7.5 cm and 11 cm in place of 2 cm and 8 cm respectively for 45°
16. Repeat steps 2 to 13 using 13 cm and 15 cm in place of 2 cm and 8 cm respectively for 60°

Fig. 4: 0° roof structure model

Fig. 5: 15° roof structure model

Fig. 6: 30° roof structure model

Fig. 7: 45° roof structure model

Fig. 8: 60° roof structure model

B. Method of Experimentation:

I. Materials Needed: Listed are materials needed for experimentation:

- Nylon String
- Scissor
- 2 Tables with Equal height
- Plastic Bucket
- Sand
- Plastic Cup/Plastic Bottle
- Scale
- Weights
- Ruler

- Roof Structure Models (10 each): 0°, 15°, 30°, 45°, 60°)

II. Method:

1. Gather all materials
2. Set 2 tables 13 cm apart so that the roof structure model rests on its edges
3. Cut 90 cm of nylon string with a scissor
4. Tie the ends of the nylon string to each ends of the plastic bucket
5. Drape the string over the groove of the roof structure, make sure the plastic bucket is balanced
6. Place two 1 kg weights on each ends of the bucket while keeping the bucket balanced
7. Place a plastic cup in the middle of the plastic bucket
8. Carefully add sand in to the plastic cup
9. Continue adding sand until the roof structure model breaks
10. Take the entire plastic bucket, including the weights and the sand, and measure its mass with a scale
11. Record the mass
12. Repeat steps 4 to 11 with all the roof structure models

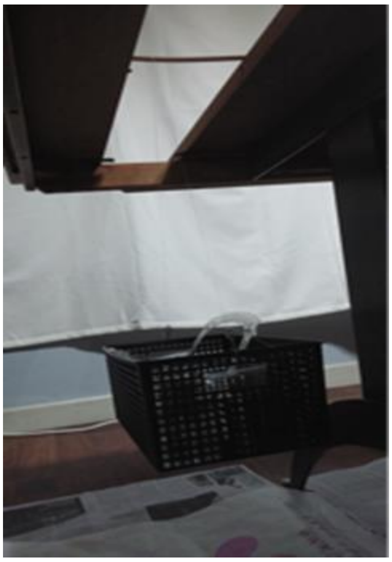


Fig. 9: 0° roof structure holding up the plastic bucket by a nylon string

C. Justification of Method:

The above method was chosen as a result of several considerations. First, the roof structure model was created because a real life sized roof structure could not be constructed. Second, only one section of the roof frame was used to ensure that the force acted only on the sloped rafters. Thirdly, sand was used to acquire the most accurate break mass. Lastly, creating only a section of the roof frame significantly saved time. Further justification of the following method can be found in Experimental Design section.

D. Limitations and Uncertainties:

Creating a real life sized roof for experimentation was too time consuming and beyond high school student's skills. Therefore, scaled down roof structure models were created. As a result, inevitable limitations were present which lowered the reliability and accuracy of the results. The results were limited due to the following factors: roof structure, experimentation, and measuring instrument.

The roof structure itself is limitation due to the material used, size of roof structure, and human error during creation. The material is a limitation because 0.1 cm thick and 1 cm wide pine wood strips were used. The pinewood is significantly smaller and thinner than the real woods used in roof constructions. To bring real meaning to the results, the results would have to be scaled up. But even if the results were scaled up, the accuracy and reliability is very low as real woods are composed of different material —ranging from softwoods like cedar and cypress to hardwoods like oak and walnut— and have different thickness to width ratio [1]. Also, using superglue to glue the wood piece together is another material limitation. The superglue is strong but it isn't how real roofs are put together. Real roofs are put together with big nails and bracings. The size of the roof structure is a limitation because the roof structure model is significantly smaller than real life roof structures. Similar to scaling up the pine wood material, scaling up the model structure only theoretically shows the relationship between the slope and its live load, lowering the reliability and accuracy once again. Human error is another limitation. Human error was present when sawing the wood pieces, gluing individual wood pieces together, and sandpapering ends of wood pieces. The wood pieces weren't sawed cleanly, sometimes leaving excess wood. The individual pieces weren't glued perfectly either, leaving excess glue and imperfect edges. Smoothing the edges with sandpaper resulted human error as well. The edges weren't smoothed evenly nor straight due to sandpapering by hand. Hence, such limitations of material, size, and human error produced limitations in the roof structure.

The experimentation was subject to limitation due to the overall experimentation method. Using the sand and the duration of load resting on the roof contributed to the limitation of experimentation. The sand is a limitation because the sand bucket spilled due to strong impact with the ground when the roof structure broke, reducing the accuracy of the break mass. The duration of load resting on the roof structure was inconsistent throughout the experiment. The sand was poured continuously in to the bucket with no pauses. But when the sand filled up the entire bucket, more sand needed to be added. In that process, the roof structure had to support the load longer, increasing the duration. Sometimes the structure broke in the midst of adding sand while some cases the structure broke during the pause to get more sand.

The measuring experiment served as a limitation due to its uncertainties. The digital was used to measure the break mass. The digital scale was quite accurate in that it measured up to two decimal points. However, uncertainty is still present. This particular digital scale had an uncertainty of ±0.01 g.

Despite these limitations, the roof structure, experimentation, as well as measuring devices were created and used to the best of its abilities. The results, therefore, are still significant and reliable to some extent.

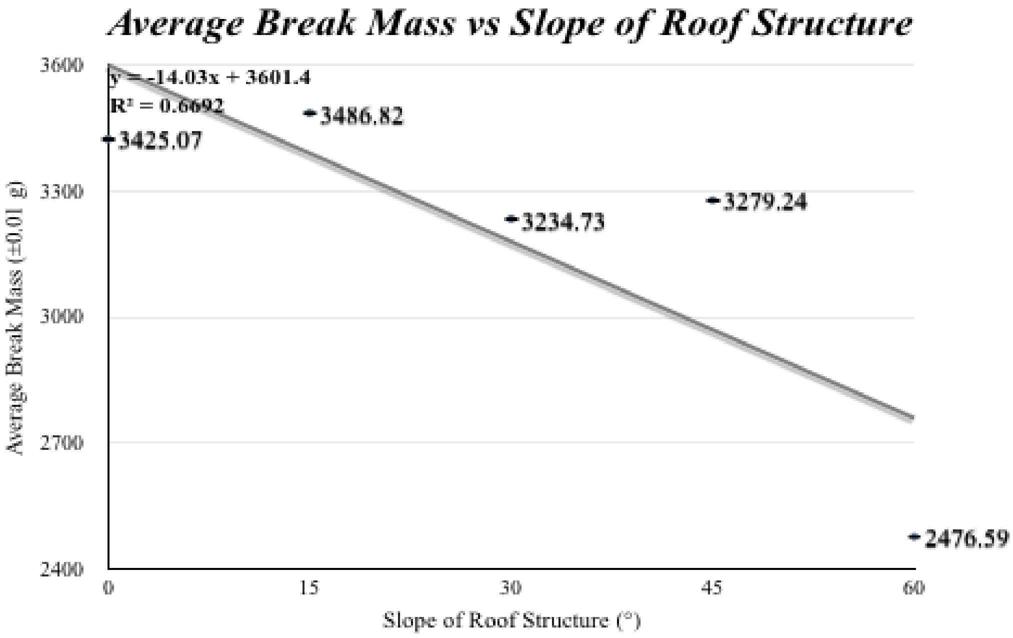
IV. RAW AND PROCESSED DATA

TABLE I: Mass of Roof Structure Models

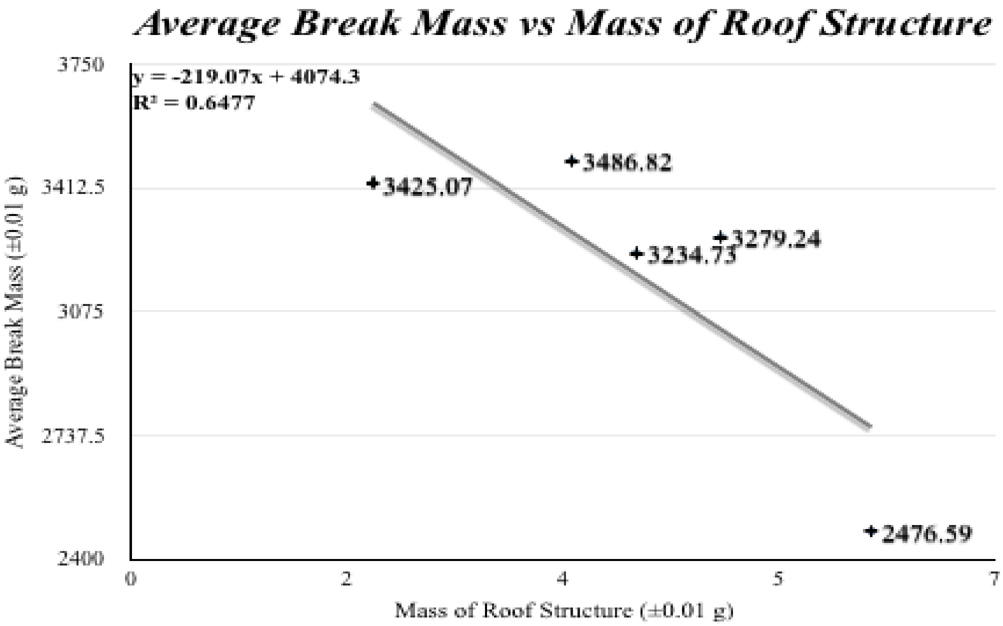
Slope of Roof		**0°**	**15°**	**30°**	**45°**	**60°**
Mass of Roof Structure Models (±0.01g)	Model 1	1.89	3.74	3.99	4.73	6.09
	Model 2	1.95	3.59	4.23	4.65	5.67
	Model 3	1.94	3.68	4.04	4.79	6.07
	Model 4	2.01	3.79	4.03	4.77	5.81
	Model 5	1.89	3.24	3.96	4.82	6.03
	Model 6	2.05	3.71	3.89	4.99	5.84
	Model 7	1.88	3.74	4.18	4.93	5.99
	Model 8	1.96	3.49	4.24	4.70	6.00
	Model 9	1.96	3.60	4.11	4.73	6.15
	Model 10	2.07	3.11	4.26	4.69	6.34
Average		1.96	3.57	4.09	4.78	6.00

TABLE II: Break Mass of Roof Structure Models

Slope of Roof		**0°**	**15°**	**30°**	**45°**	**60°**
Break Mass of Roof Structure Models (±0.01g)	Model 1	3279.08	3149.71	3259.66	3328.00	3144.46
	Model 2	3341.63	3255.58	3122.86	2931.00	1999.36
	Model 3	3541.76	3432.11	2875.99	3442.51	1729.90
	Model 4	3578.96	4153.13	3293.84	3170.19	2893.94
	Model 5	3333.75	3339.39	3429.90	3002.70	2427.15
	Model 6	3644.98	4165.36	2904.50	3483.04	2800.80
	Model 7	3247.27	3834.92	3303.69	3340.15	2184.71
	Model 8	3398.53	3194.27	3563.81	3366.44	2410.70
	Model 9	3442.20	3380.72	2587.45	3172.86	2531.55
	Model 10	3442.55	2963.00	4005.62	3555.55	2643.34
Average		3425.07	3486.82	3234.73	3279.24	2476.59



GRAPH I: Average Break Mass vs Slope of Roof Structure



GRAPH II: Average Break Mass vs Mass of Roof Structure

V. ANALYSIS

A. Force Diagram:

The way in which the roof structure broke and the reasons for assuming uniform load can not be analyzed using quantitative data. Hence, a force diagram is used.

When mass was increased, only the sloped rafter of the roof structure bent. This ensured that the mass only rested on the sloped rafters and not on any other parts of the roof structure. The force diagram shows that the force acting on the roof structure is quite similar to a force on an incline. The mass of the weights and sand act like weight (mg), normal force is perpendicular to the sloped surface, and force along the sloped surface is present. Based on the force diagram, the force acting on the sloped rafter is distributed down the sloped surface and out to the tables. Thus, the weakest point —the groove— on the sloped rafter broke when the mass exceeded the roof structure’s maximum live load.

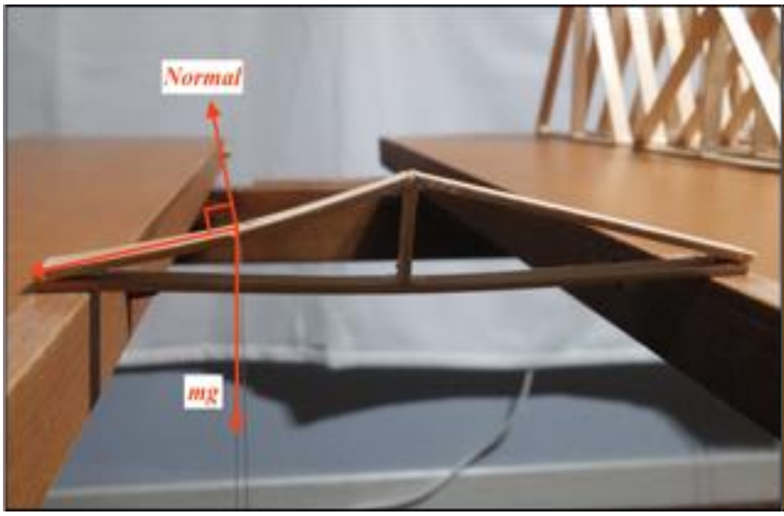


Fig. 10: Force diagram of 15° roof structure model **Fig. 11: Broken 15° roof structure model**

Uniform load was assumed because the sheer length of the wood piece holding the mass was significantly short. Because the roof structure is small, when force acted on a single point, the force was distributed throughout the wood piece, causing the whole wood piece to support the load. The force acted on the entire sloped rafter as the force was distributed and carried away to the table. In other words, the wood piece was short enough to approximate uniform load [8].

B. Quantitative Analysis:

The break mass was recorded in grams. To compare the results of the experimentation to real life numbers, the results needs to be converted to pounds per square feet (psf), a widely used unit for live load. Several assumptions and calculations involving dimensional analysis were made as a result.

i. Calculating Uniform Load:

- Because uniform load was assumed, the amount of mass acting on a particular area needed to be determined. The uniform load for the roof structures was break mass divided by the area of the wood piece in which the mass rested on. The wood piece supporting the mass was different for each slope of the roof. For instance, the 15 cm x 1 cm wood piece held the mass in 0° roof structure while the 8 cm x 1 cm wood piece held the mass in 15° roof structure.

Therefore,

$$\text{Uniform Load} = \frac{\text{Break Mass}}{\text{Area of Wood Piece holding the Mass}}$$

TABLE III: Uniform Load of Roof Structure Models in g/mm^2

Uniform Load					
Slope of Roof	0°	15°	30°	45°	60°
Uniform Load (g/mm^2)	2.3±0.1	4.4±0.2	3.6±0.2	3.0±0.2	1.65±0.09

ii. Dimensional Analysis: g/mm^2 to lb/ft$^{2:}$

- The following uniform loads in g/mm^2 were converted into pounds per square foot using dimensional analysis. 1 lb was equal to 453.592 g and 1 ft^2 was equal to 92903 mm^2.

- The results are significantly greater than real life roofs due to the materials used and conditions of the roof structure. The wood pieces were extremely small and flexible which allowed a lot of bending and distribution of force.

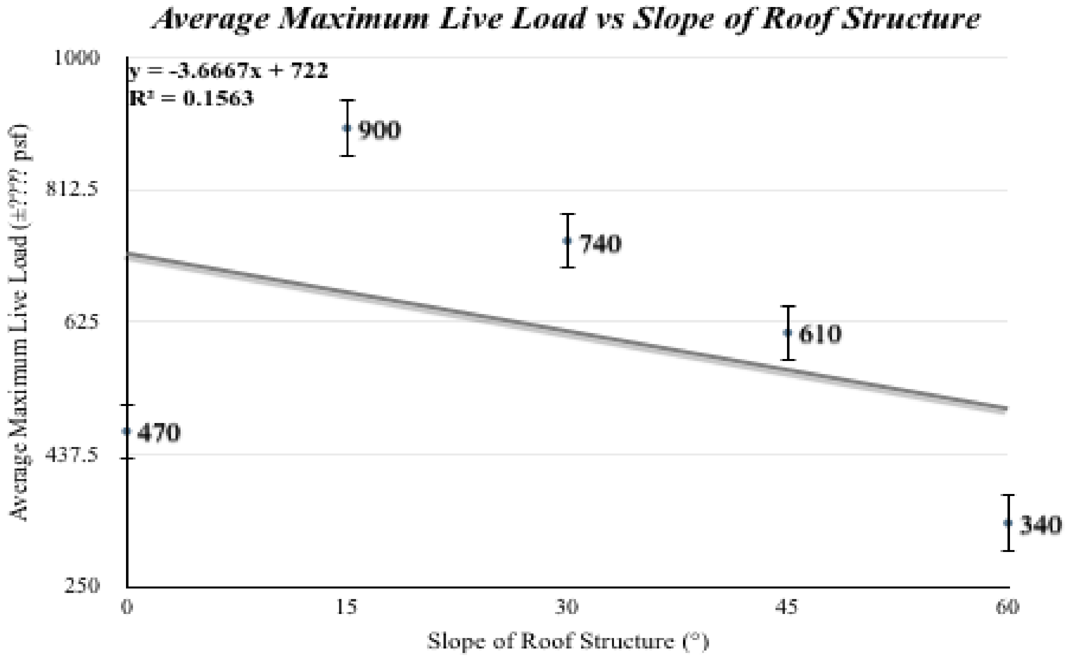
- Therefore, $1\frac{g}{mm^2} = 204.816\frac{lb}{ft^2}$ according to $\frac{1g}{mm^2} \times \frac{1lb}{453.592g} \times \frac{92903mm^2}{1ft^2} = \frac{92903}{453.592}\frac{lb}{ft^2} = 204.816\frac{lb}{ft^2}$

TABLE IV: Uniform Load of Roof Structure Models in lb/ft^2

Average Maximum Live Load of Roof Structures in lb/ft^2 (psf)					
Slope of Roof	0°	15°	30°	45°	60°
Average Maximum Live Load (lb/ft^2)	470±20	900±40	740±40	610±40	340±20

iii. Graph of Average Maximum Live Load in psf:

- New graph with average maximum live load in psf instead of average break mass was created in order to best visualize the relationship between slope and the maximum live load.



GRAPH III: Average Maximum Live Load of Roof Structures in psf vs Slope of Roof Structure Models

According to the initial tables and graphs involving average break mass of roof structures, the difference between 0° and 15° was not as significant to determine the optimal slope of the roof. The results and trendline clearly showed that the ability to withstand mass decreased as the slope increased, but the difference between the average break mass of 0° and 15° was too small to tell whether one was stronger than the other.

However, as the average break mass value was converted to average maximum live load in psf, the difference between the two became clearly visible. As uniform load was assumed, the sheer area supporting the loads differed between the 0° and 15° roof structures. Revealing that the 15° roof structure supported significantly more amount of load per area than the 0° roof structure. 0° roof structure supported less load than 30° and 45° roof structures as well.

The trendline still supports they idea that as the slope of roof increase, its maximum live load decreases, but it only applies to slopes after 0°. The average maximum live load of 15° roof structure was the highest with 892.70 psf.

The trendline also suggests that the average maximum live load continues to increase as slope gets close to 0°. Hence, the optimal slope of roof will be between 0° and less than or equal to 15° — $\text{Optimal Slope} = 0^\circ < x \le 15^\circ$ — according to the data collected and analyzed as maximum live load decreases as slope of roof increases after 0°.

VI. CONCLUSION

Due to the experimentation and many assumptions, limitations, and uncertainties, the experiment is only able to predict that the optimal slope of roof is $0^\circ < x \le 15^\circ$. However, despite the broad range of optimal slopes, the experiment is still able to show that the average maximum live load decreases as the slope of roof increases after 0°.

The hypothesized slope of 0° supported less live load than 15°, 30°, and 45° with 467.67 psf. Therefore, the hypothesis that the flat roof will support the most live load is not justified through this investigation. However, the hypothesis that as the slope of the roof increases, the amount of load it can with stand decreases is justified as slopes increasing after 0° supported less live load than the previous slopes.

The results of this investigation are only applicable to gable roofs as other roof shapes have different structures. But it can still be used to construct stronger, safer gable roofs to prevent roof collapsing in the future. Moreover, the investigation can be refined either to determine a more accurate optimal slope of roof by investigating slopes between 0° and 15° with narrower intervals or to determine the effect of the number of webs in truss roofs on its maximum live load.

VII. APPENDIX

A. Diagram with Specific Terms indicating parts of the Roof Structure:

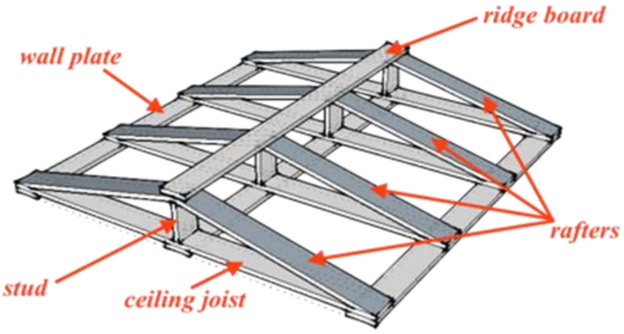


Fig. 12: Diagram of Roof with Terminologies

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